

Exercise 27

Find $f'(x)$ and $f''(x)$.

$$f(x) = (x^3 + 1)e^x$$

Solution

Use the product rule to differentiate $f(x)$.

$$\begin{aligned} f'(x) &= \frac{d}{dx} [(x^3 + 1)e^x] \\ &= \left[\frac{d}{dx}(x^3 + 1) \right] (e^x) + (x^3 + 1) \left[\frac{d}{dx}(e^x) \right] \\ &= (3x^2)(e^x) + (x^3 + 1)(e^x) \\ &= 3x^2e^x + x^3e^x + e^x \\ &= (3x^2 + x^3 + 1)e^x \end{aligned}$$

Use the product rule again to differentiate $f'(x)$.

$$\begin{aligned} f''(x) &= \frac{d}{dx} [(3x^2 + x^3 + 1)e^x] \\ &= \left[\frac{d}{dx}(3x^2 + x^3 + 1) \right] (e^x) + (3x^2 + x^3 + 1) \left[\frac{d}{dx}(e^x) \right] \\ &= (6x + 3x^2)e^x + (3x^2 + x^3 + 1)(e^x) \\ &= (6x + 6x^2 + x^3 + 1)e^x \end{aligned}$$